

Exercise 35

- (a) The curve $y = 1/(1 + x^2)$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $(-1, \frac{1}{2})$.
- (b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

Solution

Start by finding the slope of y at $x = -1$. Evaluate the derivative using the quotient rule.

$$\begin{aligned} y' &= \frac{d}{dx} \left(\frac{1}{1 + x^2} \right) \\ &= \frac{\left[\frac{d}{dx}(1) \right] (1 + x^2) - \left[\frac{d}{dx}(1 + x^2) \right] (1)}{(1 + x^2)^2} \\ &= \frac{(0)(1 + x^2) - (2x)(1)}{(1 + x^2)^2} \\ &= -\frac{2x}{(1 + x^2)^2} \end{aligned}$$

Evaluate it at $x = -1$.

$$y'(-1) = \frac{1}{2}$$

Therefore, the equation of the tangent line with slope $1/2$ that goes through $(-1, \frac{1}{2})$ is

$$y - \frac{1}{2} = \frac{1}{2}(x + 1).$$

